UKMT Team Maths Challenge

National Final

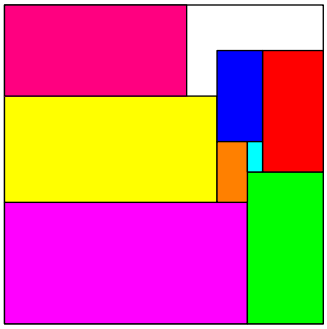
Monday 17th June 2013

Poster Round

Packing

Question 1

A *domino* consists of two squares joined together *edge to edge*. The diagram shows eight different coloured dominoes packed into a square.

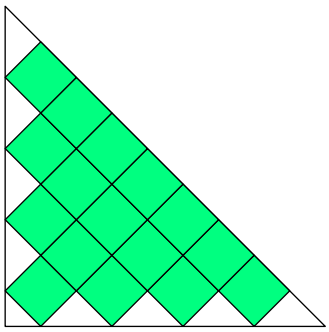


The dominoes are of size $n \times 2n$ for $n = 1$ to 8, that is, 1×2, 2×4, 3×6, $\ldots$, 8×16.

What is the density of the packing? In other words, what fraction of the area of the square is occupied by the dominoes?

Question 2

A unit square has sides of length 1. The figure shows 16 unit squares packed into an isosceles right-angled triangle.



Prove that there is a packing with density

$$\left(\frac{2n}{2n+1}\right)^2$$

of n^2 unit squares into an isosceles right-angled triangle, where n is any positive integer.

Question 3

Let T_n be the nth triangular number, so $T_1 = 1, T_2 = 3, T_3 = 6, \ldots$.

Prove that T_n circles with radius 1 may be packed into an isosceles right-angled triangle with area $2n^2 + 1 + 2n\sqrt{2}$.