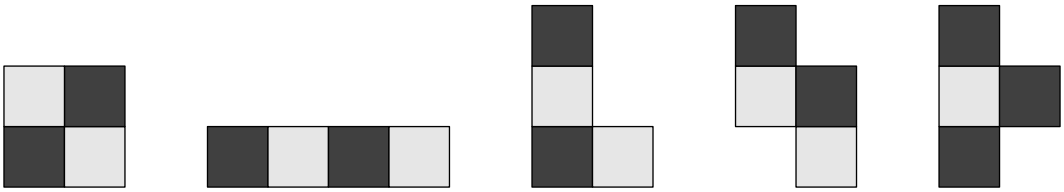
# Mathematical impossibility: questions

## Question 1

The diagram below shows the five tetrominoes, each of which is made from four connected $1 \times 1$ squares.



Prove that it is impossible to tile a rectangle, without gaps or overlaps, using all five tetronimoes once each (even allowing the shapes to be rotated or turned over).

## Question 2

A *rook* is a chess piece which moves in a straight line any number of squares along a row or column. A rook makes a *closed tour* if it visits every square exactly once, except that the final square is the same as the starting square. The rook is considered to visit every square that it passes over.

Suppose that $N$ is an odd integer greater than 1. Prove that a closed rook's tour on an $N \times N$ board is impossible.

## Question 3

(a) Prove that two regular pentagons and a regular decagon will fit together around a point.

(b) Prove that it is impossible to tile the plane using only regular pentagons and regular decagons.