Poster Competition

In addition to four rounds similar to those at the regional finals (Group Circus, Cross Number, Head to Head and Relay), there will be a Poster Competition at the National Final. All teams are required to submit a poster. This will be judged separately and will not affect the Team Maths Challenge score, but forms an integral part of the National Final.

After the competition some posters may be retained by the UKMT in order to be reproduced for promotional purposes; all the original posters will eventually be returned to schools.

On the day, teams will have 50 minutes to create a poster on a sheet of A1 paper (landscape), which will be provided. Sheets of A4 paper will also be available.

The subject of the poster will be *The golden ratio* (see overleaf). Teams must carry out research into this topic in the weeks leading up to the final.

Teams may create materials beforehand, but such prepared work must be on sheets no larger than A4 and must be assembled to create the poster on the day.

A team which arrives with a poster already assembled will be disqualified.

The materials of the poster must not extend beyond the edge of the A1 paper.

The judges will not touch the poster, so all information must be clearly visible.

Your team number (assigned to you on arrival) must be clearly visible in the bottom right-hand corner of the poster. There must be nothing else on the poster to identify the team.

Reference books may not be used at the competition, and large extracts copied directly from books or the internet will not receive much credit.

Teams must bring with them any drawing equipment they think they will need.

Glue sticks and scissors will be provided.

The content of each poster is limited only by the imagination of the team members. *However, on the day each team will be presented with three questions on the subject—the answers to these questions must be incorporated into the structure of the poster*. Teams may be asked to provide geometric or algebraic proofs, and some ingenuity may be involved.

Posters will be judged on the following criteria:

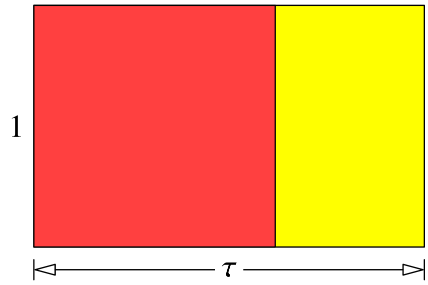
General mathematical content	(12 marks)
Imagination and presentation	(12 marks)
Answers to the questions	(24 marks)

UKMT Team Maths Challenge 2011 — Poster Competition

The golden ratio

Start with a $\tau \times 1$ rectangle, as shown, and remove the red square to leave the yellow rectangle.

What value of τ makes the yellow rectangle and the original rectangle similar—the same shape—meaning that the ratio length : width is the same for each?



> Why does τ satisfy the equation $\tau^2 = \tau + 1$?

The value of $\tau > 1$ is called the *golden ratio* (sometimes the letter ϕ is used) and a rectangle of the corresponding shape is known as a *golden rectangle*. The yellow rectangle has the same shape as the original, so it is also golden.

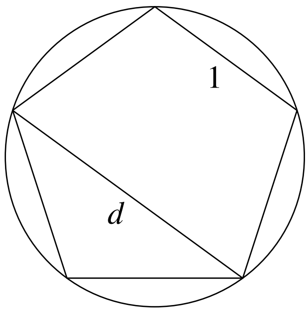
> What is the *Fibonacci tiling*?

The equation $\tau^2 = \tau + 1$ may be used to find simple expressions for other powers of τ. To find τ^3, for example, multiply both sides of the equation by τ to obtain $\tau^3 = \tau^2 + \tau$. Now substitute for τ^2 to get $\tau^3 = (\tau + 1) + \tau = 2\tau + 1$.

> Is there a simple expression for τ^{-1}, that is, $\dfrac{1}{\tau}$?

The golden ratio arises in the regular pentagon, as the ratio of the lengths of a diagonal and a side.

This result may be proved in various ways. Let each side have length 1 and each diagonal length d. By using similar triangles or Ptolemy's theorem for example, one may show that $d^2 = d + 1$, so that d is the golden ratio τ.



> Where do golden rectangles appear in the dodecahedron?

Unfortunately, it is possible to get carried away with fanciful ideas about the golden ratio. The following article describes what may go wrong and may help you to avoid including fictional or incorrect information on your poster.

Keith Devlin. *Good stories, pity they're not true*. Devlin's Angle. Mathematical Association of America. June 2004. URL: `http://www.maa.org/devlin/devlin_06_04.html`