Instructions

In addition to four rounds similar to those at the regional finals (Group Circus, Crossnumber, Shuttle and Relay), there will be a Poster Competition at the National Final. All teams are required to submit a poster. This will be judged separately and will not affect the Team Maths Challenge score, but forms an integral part of the National Final.

After the competition some posters may be retained by the UKMT in order to be reproduced for promotional purposes.

On the day, teams will have 50 minutes to create a poster on a sheet of A1 paper (landscape), which will be provided. Sheets of A4 paper will also be available.

The subject of the poster will be *Mathematical billiards* (see overleaf). Teams must carry out research into this topic in the weeks leading up to the final.

Teams may create materials beforehand, but such prepared work must be on sheets no larger than A4 and must be assembled to create the poster on the day.

A team which arrives with a poster already assembled will be disqualified.

The materials of the poster must not extend beyond the edge of the A1 paper.

The judges will not touch the poster, so all information must be clearly visible.

Your team number (assigned to you on arrival) must be clearly visible in the bottom right-hand corner of the poster. There must be nothing else on the poster to identify the team.

Reference books may not be used at the competition, and large extracts copied directly from books or the internet will not receive much credit.

Teams must bring with them any drawing equipment they think they will need.

Glue sticks and scissors will be provided.

The content of each poster is limited only by the imagination of the team members. However, on the day each team will be presented with three questions on the subject—*the answers to these questions must be incorporated into the structure of the poster*. Teams may be asked to provide geometric or algebraic proofs, and some ingenuity may be involved.

Posters will be judged on the following criteria:

General mathematical content	12 marks
Imagination and presentation	12 marks
Answers to the questions	24 marks

Mathematical billiards

In *mathematical billiards*, we have one ball on a table, and the ball moves with constant speed forever (we pretend that there is no friction to slow the ball down). The rules of motion are as follows: a ball will move in a straight line until it hits the wall of the table; the ball will then bounce off the wall obeying the rule "angle of reflection equals angle of incidence". There are no pockets for the ball to fall into, though the ball is assumed to stop if it ever reaches a corner.

For different shapes of table, what happens to the ball as it continues to bounce off the walls?

What happens when the table is an $m \times n$ rectangle, where m and n are integers, and the ball is projected from a corner at 45° to a side?

If the ball retraces the same path for ever after a finite number of bounces, then its path is said to be *periodic*. *Unfolding* is a useful technique for investigating periodic paths when the table is a polygon. Instead of considering the path of the ball on the original table, whenever the ball reaches an edge, reflect the table in that edge and let the path continue in a straight line, but now on the reflected table.

For example, suppose the ball is projected at 45° towards the midpoint of an edge of a square table. Then the ball retraces the same path after 4 bounces, as shown in Figure 1. The unfolded path appears in five squares (the original table and four copies) before returning to the starting point, as shown in Figure 2.

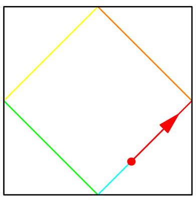


Figure 1

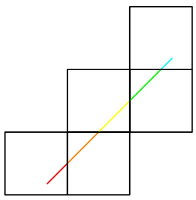
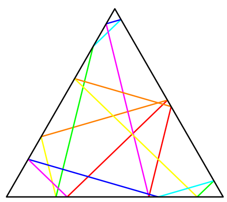


Figure 2

The figure shows a periodic path on a table which is an equilateral triangle.

What does the unfolded path look like?

How many times does the ball bounce as it travels once around the path?



How does mathematical billiards generalise to three dimensions?

Suppose a billiard ball bounces off the walls of a cube. Are there any periodic paths? What about a regular tetrahedron?