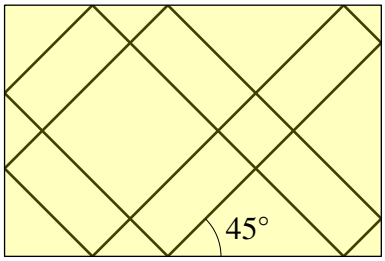
Mathematical billiards

Question 1

The diagram shows a periodic path of a billiard ball on a rectangular table measuring 3 m × 2 m.

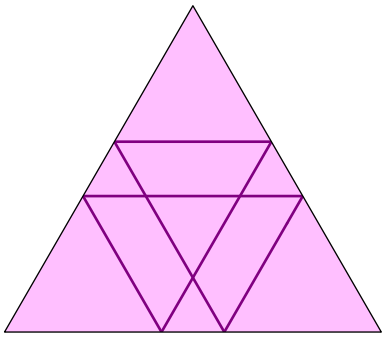
What is the length of the path?



Question 2

The diagram shows a periodic path of a billiard ball on a table that is an equilateral triangle. The ball bounces six times; between bounces the ball travels parallel to a side of the triangle.

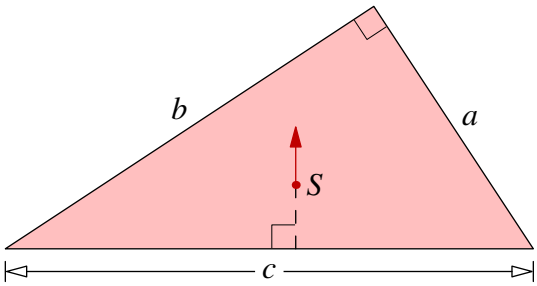
Prove that all such paths have the same length.



Question 3

A billiard table has the shape of a right-angled triangle with sides of length a, b and c, as shown.

A billiard ball is projected perpendicular to the hypotenuse from a point S inside the triangle. The ball does not hit the right-angled corner of the triangle.



Prove that the ball is once more at S, moving in its initial direction, after travelling a distance

$$\frac{4ab}{c}.$$