



Leonhard Euler and his Mathematics

Question 1

Draw a planar graph with six vertices and nine edges. Each vertex should be of degree three.

Question 2

Show that it is possible to arrange four regiments $\{A, B, C, D\}$, each consisting of four officers, each of a different rank $\{1, 2, 3, 4\}$, in a 4×4 grid, such that no regiment or rank is repeated in any row or column.

Here is a solution for three regiments, each consisting of three officers, each of a different rank.

| | | |
|------|------|------|
| $C2$ | $A3$ | $B1$ |
| $B3$ | $C1$ | $A2$ |
| $A1$ | $B2$ | $C3$ |

Question 3

Buckminsterfullerene is a form of carbon with chemical formula C_{60} . Its molecules, often known as *buckyballs*, are made of sixty carbon atoms each. The atoms in a buckyball form a *polyhedron*, with an atom at each vertex. Each vertex of the polyhedron is surrounded by exactly two regular hexagons and exactly one regular pentagon and no other polygons.

- How many edges and faces does the polyhedron have?
- How many of the faces of the polyhedron are hexagons?

You must show all your working for each part.